Are American rivers Tokunaga self-similar? New results on fluvial network topology and its climatic dependence

Supplementary Online Material

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1 Introduction

This document reports supplementary analyses and data to complement the study reported in the main text. Tables 1 to 18 illustrate the effect of the area thrshold by reporting the results of the topological analysis for different area thresholds spanning over an order of magnitude. Table 19 reports the Pearson correlation coefficients for the climatic, topographic and topological variables analyzed. Table 20 summarizes the characteristics of the sites considered.

Area threshold $= 0.05\ km^2$

Table 1: Number of self-similar (SS) and Tokunaga self-similar (TSS) networks.

	# total	**# SS**	**# TSS**	**% SS**	**% TSS**
Order 6	549	525	451	96	86
Order 7	137	132	118	96	90
Order 8	39	37	30	95	81
Order 9	5	5	4	100	80
All orders	730	699	603	96	86

Table 2: Mean and standard deviation (between parentheses) of the Tokunaga parameters $\hat{a}$ and $\hat{c}$.

	$\hat{a}$	$\hat{c}$
Order 6	1.1(0.12)	2.7(0.44)
Order 7	1.1(0.06)	2.7(0.30)
Order 8	1.1(0.06)	2.7(0.21)
Order 9	1.1(0.01)	2.6(0.12)

Table 3: Number and percentage of networks with geometrically distributed side-branches (NGDS).

	# total	**# NGDS**	**% NGDS**
Order 6	549	547	99
Order 7	137	123	90
Order 8	39	10	26
Order 9	5	0	0
All orders	730	680	93

Area threshold $= 0.09\ km^2$

Table 4: Number of self-similar (SS) and Tokunaga self-similar (TSS) networks.

	# total	**# SS**	**# TSS**	**% SS**	**% TSS**
Order 6	327	317	259	97	82
Order 7	83	79	71	95	90
Order 8	23	22	19	96	86
Order 9	3	3	2	100	67
All orders	436	421	351	97	83

Table 5: Mean and standard deviation (between parentheses) of the Tokunaga parameters $\hat{a}$ and $\hat{c}$.

	$\hat{a}$	$\hat{c}$
Order 6	1.1(0.11)	2.7(0.42)
Order 7	1.1(0.06)	2.7(0.25)
Order 8	1.1(0.06)	2.6(0.21)
Order 9	1.1(0.01)	2.7(0.09)

Table 6: Number and percentage of networks with geometrically distributed side-branches (NGDS).

	# total	**# NGDS**	**% NGDS**
Order 6	327	323	99
Order 7	83	76	92
Order 8	23	13	57
Order 9	3	0	0
All orders	436	412	94

Area threshold $= 0.2\ km^2$

Table 7: Number of self-similar (SS) and Tokunaga self-similar (TSS) networks.

	# total	**# SS**	**# TSS**	**% SS**	**% TSS**
Order 6	195	189	160	97	85
Order 7	47	46	43	98	93
Order 8	10	10	9	100	90
Order 9	1	1	0	100	0
All orders	253	246	212	97	86

Table 8: Mean and standard deviation (between parentheses) of the Tokunaga parameters $\hat{a}$ and $\hat{c}$.

	$\hat{a}$	$\hat{c}$
Order 6	1.1(0.12)	2.7(0.38)
Order 7	1.1(0.08)	2.7(0.26)
Order 8	1.1(0.04)	2.6(0.13)
Order 9	—	—

Table 9: Number and percentage of networks with geometrically distributed side-branches (NGDS).

	# total	**# NGDS**	**% NGDS**
Order 6	195	195	100
Order 7	47	41	87
Order 8	10	4	40
Order 9	1	0	0
All orders	253	240	95

Area threshold $= 0.3\ km^2$

Table 10: Number of self-similar (SS) and Tokunaga self-similar (TSS) networks.

	# total	**# SS**	**# TSS**	**% SS**	**% TSS**
Order 6	150	148	130	99	88
Order 7	38	36	30	95	83
Order 8	6	6	5	100	83
Order 9	1	1	0	100	0
All orders	195	191	165	98	86

Table 11: Mean and standard deviation (between parentheses) of the Tokunaga parameters $\hat{a}$ and $\hat{c}$.

	$\hat{a}$	$\hat{c}$
Order 6	1.1(0.12)	2.7(0.35)
Order 7	1.1(0.05)	2.6(0.19)
Order 8	1.1(0.04)	2.6(0.07)
Order 9	—	—

Table 12: Number and percentage of networks with geometrically distributed side-branches (NGDS).

	# total	**# NGDS**	**% NGDS**
Order 6	150	149	99
Order 7	38	33	87
Order 8	6	4	67
Order 9	1	0	0
All orders	195	186	95

Area threshold $= 0.4\ km^2$

Table 13: Number of self-similar (SS) and Tokunaga self-similar (TSS) networks.

	# total	**# SS**	**# TSS**	**% SS**	**% TSS**
Order 6	123	121	106	98	87
Order 7	30	29	28	97	97
Order 8	5	5	4	100	80
Order 9	1	1	0	100	0
All orders	159	156	138	98	88

Table 14: Mean and standard deviation (between parentheses) of the Tokunaga parameters $\hat{a}$ and $\hat{c}$.

	$\hat{a}$	$\hat{c}$
Order 6	1.1(0.12)	2.7(0.36)
Order 7	1.1(0.06)	2.6(0.22)
Order 8	1.1(0.05)	2.7(0.05)
Order 9	—	—

Table 15: Number and percentage of networks with geometrically distributed side-branches (NGDS).

	# total	**# NGDS**	**% NGDS**
Order 6	123	122	99
Order 7	30	28	93
Order 8	5	4	80
Order 9	1	0	0
All orders	159	154	97

Area threshold $= 0.5\ km^2$

Table 16: Number of self-similar (SS) and Tokunaga self-similar (TSS) networks.

	# total	**# SS**	**# TSS**	**% SS**	**% TSS**
Order 6	108	105	89	97	85
Order 7	25	25	25	100	100
Order 8	4	4	3	100	75
Order 9	1	1	0	100	0
All orders	138	135	117	98	87

Table 17: Mean and standard deviation (between parentheses) of the Tokunaga parameters $\hat{a}$ and $\hat{c}$.

	$\hat{a}$	$\hat{c}$
Order 6	1.1(0.11)	2.7(0.37)
Order 7	1.1(0.06)	2.6(0.22)
Order 8	1.1(0.05)	2.7(0.06)
Order 9	—	—

Table 18: Number and percentage of networks with geometrically distributed side-branches (NGDS).

	# total	**# NGDS**	**% NGDS**
Order 6	108	108	100
Order 7	25	23	92
Order 8	4	3	75
Order 9	0	0	0
All orders	138	134	97

Table 19: Pearson correlation coefficients for the climatic, topographic and topological variables considered in the analysis.

	Mean Ann. Rain	Rain Event Frequency	Rain Event Duration	Mean Elevation	Std. Dev. Elevation	Max Elevation	Mean Slope	Std. Dev. Slope	Max Slope	c (R_0=0.8)	c (R_0=0.9)	c (R_0=0.98)
Mean Ann. Rain	1	-0.29 (0.041)	0.45 (0.001)	-0.29 (0.039)	0.18 (0.203)	0.07 (0.612)	0.33 (0.019)	0.32 (0.024)	0.20 (0.174)	0.51 (0.001)	0.53 (0.004)	0.58 (0.037)
Rain Event Frequency		1	-0.80 (0.000)	-0.41 (0.003)	-0.61 (0.000)	-0.64 (0.000)	-0.61 (0.000)	-0.64 (0.000)	-0.52 (0.000)	-0.41 (0.012)	-0.51 (0.005)	-0.51 (0.072)
Rain Event Duration			1	0.06 (0.657)	0.27 (0.055)	0.33 (0.017)	0.39 (0.005)	0.44 (0.001)	0.38 (0.007)	0.40 (0.013)	0.57 (0.002)	0.59 (0.033)
Mean Elevation				1	0.69 (0.000)	0.84 (0.000)	0.55 (0.000)	0.63 (0.000)	0.66 (0.000)	0.00 (0.997)	-0.15 (0.432)	-0.24 (0.437)
Std. Dev. Elevation					1	0.89 (0.000)	0.82 (0.000)	0.84 (0.000)	0.81 (0.000)	0.28 (0.095)	0.24 (0.219)	0.13 (0.674)
Max Elevation						1	0.74 (0.000)	0.82 (0.000)	0.82 (0.000)	0.22 (0.194)	0.15 (0.454)	0.12 (0.693)
Mean Slope							1	0.93 (0.000)	0.83 (0.000)	0.29 (0.072)	0.24 (0.926)	0.03 (0.925)
Std. Dev. Slope								1	0.93 (0.000)	0.30 (0.065)	0.24 (0.218)	0.11 (0.731)
Max Slope									1	0.29 (0.073)	0.26 (0.189)	0.12 (0.695)

Table 20: Physical characteristics of the analyzed catchments. Site indices refer to those reported in Fig. 7 (main text).

Site index	USGS site code	State	Outlet latitude	Outlet longitude	Horton-Strahler order	Mean Annual Rain [mm]	Mean rain frequency [d^{-1}]	Mean rain duration [d]
1	1610000	WV-MD	39.5389	-78.4567	8	947	0.07	11.1
2	9292500	UT	40.5119	-110.3408	7	725	0.12	3.7
3	14321000	OR	43.5861	-123.5542	9	1283	0.08	8.2
4	5517500	IN	41.2214	-86.9644	8	957	0.13	5.5
5	14233400	WA	46.4703	-122.0975	7	1993	0.07	9.7
6	1548500	PA	41.5217	-77.4478	7	949	0.12	6.2
7	14101500	OR	45.2417	-121.0939	7	800	0.09	7.1
8	3179000	WV	37.5439	-81.0106	7	969	0.12	6.2
9	14232500	WA	46.4417	-121.8628	6	2044	0.09	8.1
10	1500500	NY	42.3214	-75.3169	7	1029	0.08	9.9
11	12149000	WA	47.6661	-121.9242	7	2380	0.08	9.1
12	14080500	OR	44.1139	-120.7944	8	359	0.10	5.7
13	3198500	WV	38.1797	-81.7117	6	1135	0.12	6.0
14	5508000	CA	39.525	-120.9369	7	1718	0.10	4.6
15	5412500	WA	48.3653	-120.115	8	916	0.10	6.5
16	7163000	IN	38.2375	-86.2283	7	1121	0.14	4.4
17	11355500	MO	37.7592	-92.4517	8	988	0.14	4.3
18	5410490	IL	42.1958	-88.9986	8	892	0.13	5.0
19	7144200	IN	40.7819	-86.2639	7	944	0.13	5.2
20	1558000	NE	42.4597	-103.1708	7	381	0.12	4.9
21	6454500	CA	38.3797	-119.4492	7	832	0.11	4.5
22	3075500	LA	30.6403	-92.8139	9	1480	0.12	5.7
23	12459000	MS	32.3264	-88.9097	8	1397	0.13	4.8
24	3050500	MI	42.7506	-84.5553	7	794	0.13	5.2
25	12413500	NM	33.0611	-108.5367	8	537	0.10	5.5
26	1663500	TX	31.0817	-98.0164	7	741	0.12	4.3
27	3020500	KS	37.0036	-96.315	7	878	0.14	3.7
28	1667500	SC	34.4444	-82.1764	7	1246	0.12	5.4
29	4212000	WI	43.5742	-90.6431	7	830	0.15	3.8
30	3024000	MT	45.1119	-110.7936	8	710	0.07	10.5
31	1348000	ID	46.1506	-115.5864	7	1291	0.10	6.7
32	3175500	ID	47.5722	-116.2528	8	1179	0.09	7.4
33	6334500	MO	39.0164	-90.9833	8	938	0.13	4.8
34	4185000	OR	42.4472	-121.2375	8	552	0.12	3.9
35	1514000	WA	46.7761	-123.0344	7	1579	0.07	9.8
36	1674500	TX	28.2917	-97.2789	7	837	0.12	4.8
37	4201500	AL	34.6242	-86.3064	8	1486	0.14	4.5
38	2030500	PA	40.0822	-76.7203	7	1015	0.15	4.6
39	3155500	AR	35.5769	-94.0153	6	1251	0.14	4.2
40	7252000	OH	40.5147	-82.1956	7	926	0.14	5.1
41	2143040	GA	31.2211	-81.8675	8	1222	0.11	6.4
42	6817000	WA	47.1514	-121.9486	6	1882	0.08	9.2
43	7261000	CA	36.4067	-118.9533	7	725	0.10	3.3
44	11210500	CA	33.1069	-116.8653	6	526	0.10	3.0
45	7072000	IA	43.0625	-92.6731	8	837	0.13	4.9
46	7346050	ME	44.7072	-69.9392	9	1179	0.16	3.9
47	7375500	FL	30.5539	-84.3842	7	1314	0.11	5.8
48	2118000	NE	40.3328	-98.0667	7	658	0.12	5.0
49	3438000	WY	43.2425	-109.0097	8	487	0.10	6.6
50	2016000	NM	35.4028	-104.4431	6	374	0.12	4.0